Mathematical Structuralism and the Univalent Foundations

Min Cheol Seo

Mathematicians and formalisation practitioners routinely treat isomorphic structures as "the same", yet our standard foundations do not. In set theory, different reductions of arithmetic (von Neumann, Zermelo, etc.) disagree on sentences like "$1 \in 3$", even though they are equally good models of the Peano axioms. In proof assistants, different encodings of $\mathbb{N}$, or different implementations of groups and categories, are handled as distinct unless we explicitly transport along *isomorphisms*. These are all instances of so-called the *Benacerraf Problem*: if there are many mutually isomorphic realisations, what—if anything—are the mathematical objects over and above the structure?

In philosophy, *mathematical structuralism* is the thesis that mathematical objects are positions in abstract structures that exist independently of their set-theoretic or computational realisations. Representations related by isomorphism are equally good instantiations of the same *ante rem* structure; facts that vary across such instantiations are not genuinely about that structure.

This viewpoint imposes a constraint on foundations: identity at the foundational level should track structural equivalence, not encoding. The Univalent Foundations (Homotopy Type Theory and the Univalence Axiom) is a live attempt to meet this demand: Types form ∞-groupoids, equality is given by paths, and univalence identifies equality of types with their equivalence. In this talk, I will explain this picture and contrast it with familiar set-theoretic (ZF(C)) and categorical foundations (ETCS).